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$$=\frac{\sqrt{2}}{r^3}\int_0^{\frac{1}{2}r\sqrt{2}}[r-x\sqrt{2}]^2dx+\frac{[2+\sqrt{2}]}{r^3}\int_0^{\frac{1}{2}r\sqrt{2}}\int_0^{\theta'}z^2dx\,d\theta$$

$$=\tfrac{1}{3}+\frac{[2+\sqrt{2}]}{r^3}\int_0^{\frac{1}{2}r\sqrt{2}}\{[r^2-2x^2][\tfrac{1}{4}\pi-\sin^{-1}\frac{x}{r}]-2x[\sqrt{(r^2-x^2)}-x]\}dx=1-\frac{\sqrt{2}}{9}.$$

MISCELLANEOUS.

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136. Proposed by **W. J. GREENSTREET, M. A.,** Editor of The Mathematical Gazette, Stroud, England.

(1) Solve (to five places) the equations, $\sin(x+\frac{1}{6}\pi)=10\sin x$, and $a\cos\phi\log\sin\phi=p$ where a is small and positive, and $\varphi=\alpha+\kappa$, where κ is very small and α is not very small; (2) If $a\theta=b\varphi$ where a is prime to b, and $\sin\theta=p$, $\sin\varphi=q$, how many values of q are there for each of p ? (3) if $2x=\sin^{-1}x$, show there is only one positive value of x, and find it.

Solution by **G. B. M. ZERR, A. M., Ph.D.,** Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

(1) $\sin(x+\frac{1}{6}\pi)=\frac{1}{2}\sqrt{3}\sin x+\frac{1}{2}\cos x=10\sin x$.

$\therefore(20-\sqrt{3})\sin x=\cos x$, or $\tan x=\dfrac{1}{20-\sqrt{3}}=\dfrac{20+\sqrt{3}}{397}=.05474$. $\therefore x=3°8'$.

$\cos\varphi\log\sin\varphi=p/a$ or $\log(\sin\varphi)^{\cos\phi}=p/a$.

$\therefore\sin^2\varphi=e^{2p\sec\phi/a}$.

$$1-\cos^2\varphi=1+\frac{2p}{a}\sec\varphi+\frac{4p^2}{2!a^2}\sec^2\varphi+\frac{8p^3}{3!a^3}\sec^3\varphi+\ldots\ldots$$

$$-a/2p=\sec^3\varphi+\frac{p}{a}\sec^4\varphi+\frac{2p^2}{3a^2}\sec^5\varphi+\frac{p^3}{3a^3}\sec^6\varphi+\ldots\ldots$$

By reversion of series,

$$\sec\varphi=-\left(\frac{a}{2p}\right)^{\frac{1}{3}}-\frac{1}{3}\left(\frac{p}{4a}\right)^{\frac{1}{3}}-\frac{p}{18a}-\frac{p^2}{81a^2}\left(\frac{a}{2p}\right)^{\frac{1}{3}}-\frac{29p^2}{2430a^2}\left(\frac{p}{4a}\right)^{\frac{1}{3}}\ldots\ldots$$

The value of $\sec\varphi$ to five terms. Since a is small, p is still smaller. When any multiple of $\frac{1}{2}\pi$ is substituted for φ, $\cos\varphi\log\sin\varphi$ is a maximum.

(2) $\sin\theta=\sin(m\pi-\pi)=\sin(2m\pi+\theta)=p$,

$\sin\varphi=\sin(m\pi-\varphi)=\sin(2m\pi+\varphi)=q$.

Since $a\theta=b\varphi$, but one value of q satisfies one value of p; for $a(n\pi\pm\theta)=b(n\pi\pm\varphi)$ only when $a=b$ and $a\theta=b(n\pi\pm\varphi)$ never.

$$(3)\ x=\sin 2x=2x-\frac{8x^3}{3\,!}+\frac{32x^5}{5\,!}-\frac{128x^7}{7\,!}+\frac{512x^9}{9\,!}+\ldots\ldots$$

$$\therefore y=0 \text{ or } 1=\frac{8x^2}{3\,!}-\frac{32x^4}{5\,!}+\frac{128x^6}{7\,!}-\frac{512x^8}{9\,!}+\frac{2048\,x^{10}}{11\,!}-\ldots\ldots$$

By reversion of series, $x=.94775$, nearly.
$2x=108^\circ\ 36'$, nearly. $x=54^\circ\ 18'$, nearly.

137. **Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.**

The first transvectant of the binary cubic and its second transvectant is the *cubicovariant* of the binary cubic.

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and W. G. GREENWOOD, A. B., Professor of Mathematics, McKendree College, Lebanon, Ill.

Let $u=ax^3+3bx^2y+3cxy^2+dy^3$, be the binary cubic. Then the second transvectant is

$$\frac{d^2u}{dx^2}\frac{d^2u}{dy^2}-\left(\frac{d^2u}{dxdy}\right)^2=0=(ac-b^2)x^2+(ad-bc)xy+(bd-c^2)y^2.$$

This is the Hessian of the cubic.
Let $v=(ac-b^2)x^2+(ad-bc)xy+(bd-c^2)y^2$.
The first transvectant of u, v is

$$\frac{du}{dx}\frac{dv}{dy}-\frac{du}{dy}\frac{dv}{dx}=0.$$

$$\therefore 3(ax^2+2bxy+cy^2)[ad-bc)x+2(bd-c^2)y]$$
$$-3(bx^2+2cxy+dy^2)[(ad-bc)y+2(ac-b^2)x]=0.$$

$$\therefore (a^2d-3abc+2b^3)x^3+3(abd-2ac^2+b^2c)x^2y$$
$$+3(2b^2d-bc^2-acd)xy^2+(3bcd-ad^2-2c^3)y^3=0.$$

This is the cubicovariant of the cubic u.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

170. **Proposed by J. F. LAWRENCE, A. B., Breckenridge, Mo.**

Suppose the market value of 5% bank stock to be $11\frac{1}{9}$% higher than 8% corporation bonds; I realize 8% on my investment, and my income from each is \$180; what did I invest in each?